

# Neutron Star Masses and Radii as Inferred from kilo-Hertz QPOs

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## ABSTRACT

Kilo-Hertz (kHz) Quasi-periodic oscillations (QPOs) have been discovered in the X-ray fluxes of 8 low-mass X-ray binaries (LMXBs) with the Rossi X-ray Timing Explorer (RXTE). The characteristics of these QPOs are remarkably similar from one source to another. In particular, the highest observed QPO frequencies for 6 of the 8 sources fall in a very narrow range: 1,066 to 1,171 Hz. This is more remarkable when one considers that these sources are thought to have very different luminosities and magnetic fields, and produce very different count rates in the RXTE detectors. Therefore it is highly unlikely that this near constancy of the highest observed frequencies is due to some unknown selection effect or instrumental bias. In this letter we propose that the highest observed QPO frequency can be taken as the orbital frequency of the marginally stable orbit. This leads to the conclusions that the neutron stars in these LMXBs are inside their marginally stable orbits and have masses in the vicinity of  $2.0M_{\odot}$ . This mass is consistent with the hypothesis that these neutron stars were born with about  $1.4M_{\odot}$  and have been accreting matter at a fraction of the Eddington limit for  $10^8$  years.

*Subject headings:* X-rays:Stars—Stars:Neutron—Binaries:General

## 1. Introduction

RXTE's large X-ray collection area (7,000 cm<sup>2</sup>) and microsecond time resolution ( $2^{-20}$ s) combined with its broad telemetry bandwidth makes it possible for the first time to systematically study fast time variability in the X-ray fluxes of a large number of galactic sources (Bradt, Rothschild, & Swank 1993). Since its launch on 30 December 1995, kHz QPOs have been discovered in the persistent fluxes of 8 LMXBs: Sco X-1 (van der Klis *et al.* 1996a), 4U 1728-34 (Strohmayer *et al.* 1996), 4U 1608-52 (Berger *et al.* 1996), 4U 1636-53 (Zhang *et al.* 1996), 4U 0614+091 (Ford *et al.* 1996), 4U 1735-44 (Wijnands *et al.* 1996), 4U 1820-30 (Smale, Zhang, & White 1996), and GX 5-1 (van der Klis *et al.* 1996d). In addition, episodic and nearly coherent oscillations have been discovered during several Type-I X-ray bursts from 4U 1728-34 with a frequency of 363 Hz (Strohmayer *et al.* 1996), during one Type-I burst from KS 1731-260 with a frequency of 524 Hz (Morgan & Smith 1996), during 3 bursts from the vicinity of GRO J1744-28 with a frequency of 589 Hz (Strohmayer, Lee, & Jahoda 1996), and during 4 Type-I bursts from 4U 1636-53 with a frequency of 581 Hz (Zhang *et al.* 1997).

Two simultaneously present kHz QPOs have been observed in 6 of the 8 sources, with 4U 1735-44 and 4U 1608-52 showing only one QPO so far. For Sco X-1 (van der Klis *et al.* 1996b), the difference in the centroid frequencies of the two QPOs changes with time (van der Klis *et al.* 1996b), whereas for 4U 1728-34 and 4U 0614+091, the differences are constant with time and with count rate (Strohmayer *et al.* 1996 and Ford *et al.* 1996). In the case of 4U 1728-34, the difference, within measurement uncertainties, is always 363 Hz, the same as the frequency of the nearly coherent oscillations observed during several bursts. In the case of 4U 1636-53, the difference, which is in the vicinity of 290 Hz, is quite different from the frequency of the nearly coherent oscillations during bursts. In the case of 4U 0614+091, the difference coincides with the centroid frequency of a third QPO which was observed during one half hour period. Similar information is not available for 4U 1820-30.

These QPOs show remarkably similar characteristics from source to source. Their centroid frequencies span a range from a low of 400 Hz (4U 0614+091, Ford *et al.* 1996) to as high as 1171 Hz (4U 1636-53, van der Klis *et al.* 1996c). The QPO centroid frequency is correlated with count rate, reminiscent of

the horizontal branch oscillations (HBOs) observed in Z sources (van der Klis 1995 and references therein), but with much larger power-law indexes. An exception to this correlation is 4U 1608-52 (Berger *et al.* 1996). The coherence levels of these QPOs are typically much higher than those of the horizontal branch QPOs of Z sources. Their Q-values ( $\nu/\Delta\nu$ ) can be as high as  $10^2$ . Their *root-mean-squared* (RMS) amplitudes range from a low at the threshold of detectability to as high as 12% in the RXTE/PCA (proportional counter array) band (2–60 keV). In particular, for every source where this information is available (4U 1728-34, 4U 1608-52, 4U 1636-53), the RMS amplitudes show strong dependence on energy. For example, in the case of 4U 1636-53, the RMS amplitude at 3 keV is only 4%, but at 20 keV is as high as 16% (Zhang *et al.* 1996).

Three rather different mechanisms have been proposed so far to explain the existence and characteristics of these QPOs. Klein *et al.* (1996) have proposed that these QPOs result from turbulence occurring in the settling mounds of the neutron star polar caps. “Photon bubbles oscillations” (PBO model hereafter) form in the accretion mounds and transport energy to the surface. Their numerical simulations indicate QPOs in the kHz range can result with fractional RMS amplitudes on the order of 1%. Several QPO frequencies have shown up in their simulations with the highest ones above 2 kHz. Titarchuk and Lapidus (1996) have proposed a second mechanism in which the kHz QPOs result from acoustic waves in the boundary region surrounding the neutron star (hereafter acoustic waves oscillations, or AWO). Miller, Lamb, and Psaltis (1996) propose a third model in which the QPO with the higher frequency is the Kepler frequency at the sonic point at which the radial inflow velocity transitions from subsonic to supersonic. The position of the sonic point is determined by the radiation forces which remove the angular momentum of the accreting matter. In this model the accretion flow is also modulated by the radiation flux from the neutron star polar caps pulsed at the neutron star spin frequency. This accretion flow modulation causes the Kepler frequency to beat with the neutron star spin frequency to produce the QPO with the lower frequency (hereafter, sonic-point model, or SPM). In this model there is a natural upper limit to the QPO frequencies, either the Kepler frequency at the stellar surface, or the frequency of the marginally stable orbit.

Due to the semi-quantitative nature of these models at the present and the insufficient details available from the currently existing data, none of these models can yet be definitively tested. In this *Letter* we point out the fact that the highest observed frequencies from 6 of the 8 sources are nearly the same:  $\sim 1,100$  Hz. Taking into account that these sources are believed to have very different luminosities and magnetic fields, we believe this fact strongly favors a beat-frequency model, such as the SPM, where Keplerian orbits play the most important role in producing the observed variability.

## 2. Highest Observed QPO Frequencies

Table 1 lists the eight LMXBs and their corresponding highest reported QPO frequency. To our knowledge there has been no systematic effort on the part of all the authors to determine the absolutely highest observed frequency in each case. Therefore the values should be taken as what can be determined from the data in a straightforward analysis. We believe any further dedicated effort to search for the highest frequencies from these sources will result in qualitatively similar numbers as in this table. Even with this caveat in mind, it is quite striking that the highest observed frequencies of six of the eight sources fall in a very narrow range: 1066 to 1171 Hz. Actually, of the six sources in this range, 5 of them fall in the range of 1130 to 1171 Hz.

We note the two exceptions: 4U 1608-52 and GX 5-1. 4U 1608-52 was observed during the decline of an outburst (Berger *et al.* 1996). There may be reasons for this exception. First, the observation may have been too short to have seen its highest QPO frequency. Its highest QPO frequency may have occurred during an earlier part of its outburst, thereby missed by RXTE observations. At any rate the observed frequency is *lower* than the putative upper limit and does not directly contradict any of the following arguments. GX 5-1 was also observed only for a short time. Most likely its highest QPO frequency has not been observed.

First, we note that all these reported observations have Nyquist frequencies much higher than 1100 Hz. Therefore it is impossible for any problem related to time resolution to have caused all the highest QPO frequency to be nearly the same for all these sources. Second, as shown in Table 1, these sources generate vastly different count rates in the RXTE/PCA de-

tectors, ranging from over 100,000 counts per second (cps) for Sco X-1 to 400 cps for 4U 0614+091. The counting statistics are vastly different from source to source. The sensitivity for detecting any QPO is rather different for a bright source like Sco X-1 from that for a faint source like 4U 0614+091. There appears to be no sensitivity-related limits that will make all the highest observed QPO frequencies to be the same. Therefore we conclude that the fact that all these highest frequencies cluster around 1100 Hz could not have been due to some unknown effects related to the measurement process, rather it appears to be a feature common and intrinsic to all these sources.

It is not obvious that there should be any absolute and natural upper limits on the QPO frequencies in the PBO and AWO models. Therefore in this *Letter* we will discuss the significance of this near constancy of the highest observed QPO frequencies in the context of a beat-frequency model, specifically, the SPM of Miller *et al.* In this model, the QPO with the higher centroid frequency is identified as the Kepler frequency at some radius, and the nearly coherent oscillations observed during Type-I X-ray bursts (and the 327 Hz oscillations observed for 4U 0649+091) are identified with the neutron star spin. The QPO with the lower centroid frequency results from beating of these two frequencies. Naturally the radius where the higher QPO frequency is generated can be identified as the inner edge of the near-Keplerian flow.

In the magnetospheric beat-frequency model (Alpar & Shaham 1985) for the HBOs observed in Z sources, the inner edge of the accretion disk is at the magnetic radius, which is determined by the competition of the magnetic stresses and material stresses, as well as the neutron star mass. In the most general form, the Kepler frequency ( $\nu_K$ ) at the magnetic radius can be expressed as

$$\nu_K \propto \dot{M}^\alpha B^\beta M_{ns}^\gamma, \quad (1)$$

where  $\dot{M}$  is the accretion rate,  $B$  the magnetic field, and  $M_{ns}$  the mass of the neutron star. The power law indices  $\alpha$ ,  $\beta$ , and  $\gamma$  depend on the physical conditions relevant for the accretion disk. Detailed modeling by Ghosh and Lamb (1991) has shown that  $\alpha$  range from 0.22 to 2.5,  $\beta$  from -1.2 to -0.76, and  $\gamma$  from -1.0 to 0.07 under various physical assumptions. Barring any presently unknown correlations between any of the three variables, in order to have the kind of similar Kepler frequencies at the magnetic radii for different sources, the accretion rates, and magnetic fields have

to be substantially similar for these sources. For example, in order for Sco X-1 and 4U 1728-34 to have their Kepler frequencies within 20 Hz of each other as shown in Table 1, their accretion rates would have to be within 10% of each other. This is contrary to the current understanding that Sco X-1 is believed to be accreting at or very near the Eddington limit and that 4U 1728-34 at 10% or less of the Eddington limit. Similar arguments apply to the magnetic fields of these sources. Therefore we conclude that it is unlikely that the constancy of the highest observed QPO frequencies from source to source is due to the possibility that the edges of the accretion disks, as determined by the magnetospheres of these sources, are at the same radius.

Miller and Lamb (1993) pointed out that in weakly magnetized neutron star systems, the accretion disk flow can become significantly non-Keplerian near the star because radiation forces remove angular momentum from the accreting matter. This scenario is used by Miller, Lamb, and Psaltis (1996) to propose the sonic-point model for the kHz QPOs. The sonic point is where the radial inflow velocity transitions from subsonic to supersonic and determined by the luminosity of the source, which is used by them to explain the kHz QPO frequency and count rate correlation.

As pointed out by Miller, Lamb, and Psaltis (1996), in the sonic point model, there is a natural upper limit for the kHz QPO frequency. It results because either the Keplerian disk extends all the way to the neutron star surface or that it changes from Keplerian motion to nearly free fall at the marginally stable orbit. We think it is unlikely that the highest QPO frequencies are the Kepler frequencies at the stellar surface, because, if it were so, one would have to assume the boundary layers of these sources have substantially the same size and structure despite the differences in their magnetic fields and accretion rates. In addition, the sonic point model depends on the modulated radiation fluxes from the magnetic poles to generate the lower frequency QPO. If the accretion disk were to terminate at the stellar surface, one should expect the lower frequency QPO not to be observed at the same time when the highest QPO frequency is observed. This is contrary to what has been observed in 4U 1728-34 (Strohmayer *et al.* 1996) and 4U 1636-53 (Zhang *et al.* 1997).

Therefore we believe the simplest and most straightforward explanation for the near constancy from source to source of the highest observed QPO frequencies is

that they are the orbital frequencies of the marginally stable orbits of their respective neutron stars. If true, this leads to two conclusions. First, the neutron stars are inside their marginally stable orbits. Second, the masses of these neutron stars can be determined from the simple formula (see, e.g., Shapiro & Teukolsky 1983)

$$M_{ns}/M_{\odot} = 2198/\nu_{max}. \quad (2)$$

For a typical  $\nu_{max}$  of 1100 Hz, one gets  $M_{ns} = 2 M_{\odot}$ . Here we have ignored the spin of the neutron star, which can increase (assuming the disk corotates with the star) the estimated mass by up to 10 – 15% depending on the equation of state and spin rate (see Miller & Lamb 1996).

In the next section we discuss the plausibility of these conclusions in the context of existing literature on neutron star masses and radii and of the evolution scenarios of LMXBs.

### 3. Discussion

If our interpretations are correct, we have reached two conclusions: (1) the neutron stars in LMXBs are inside their marginally stable orbits; and (2) their masses are of the order of  $2M_{\odot}$ .

Currently there has been no definitive observational information on the radii of neutron stars. Meszaros and Riffert (1987), synthesizing calculations of general relativistic effects in the beaming, spectrum, and pulse properties of accreting neutron stars, and taking account of models for X-ray pulsars and QPOs observed in X-ray pulsars and LMXBs, argue that a reasonable value for the neutron star radius in these systems is  $4GM/c^2$ . Note that the marginally stable orbit has a radius of  $6GM/c^2$  for a non-rotating star. Kluzniak and Wilson (1991) argue that hard X-ray emission from a weakly magnetic neutron star system can result if one assumes that the neutron star is smaller than its marginally stable orbit, making it possible for accreting matter to free-fall to the neutron star surface. The accreting matter will form a hot, optically thin equatorial belt with a temperature well above 100 keV. The fact that hard X-rays have been observed from LMXBs indicate that this model is plausible (see, e.g., Barret & Vedrenne 1994).

It is worth noting that nearly all the neutron star models reviewed by Baym and Pethick (1979) require that a stationary neutron star with mass above  $1.7M_{\odot}$  be inside its marginally stable orbit. This is even true for moderately rotating stars with most

of the equations of state (Friedman, Ipser, & Parker 1986). Therefore we think that in both the context of theoretical models and observational evidence, it is not unreasonable to conclude that a neutron star is inside its marginally stable orbit.

Masses of neutron stars have been measured for a few radio pulsars and X-ray pulsars (see Nagase 1989 for a review). All these mass measurements are consistent with  $1.4M_{\odot}$  with Vela X-1 being possibly more massive by  $0.2M_{\odot}$ . It is generally believed that the neutron stars in these pulsars are much younger than those in LMXBs. The apparent difference between our estimate of  $2M_{\odot}$  is not necessarily in conflict with any of these measurements.

Estimates for neutron star parameters in LMXBs have come from energy spectroscopic studies of Type-I X-rays bursts (see Lewin, van Paradijs, & Taam 1993 for a review). The results from these studies typically favor soft equations of state, thereby favoring a lower mass than  $1.4M_{\odot}$  for the neutron stars. In general, however, these estimates are plagued by systematics, such as uncertainties of source distances, relation between effective and color temperatures, the composition of the accreted matter, etc. Ebisuzaki (1987) in a particular study of bursts from 4U 1636-536 estimates the neutron star mass to be  $1.8$  to  $2.0M_{\odot}$ , in substantial agreement with our estimate in this *Letter*. However, his estimate depends on a controversial interpretation of an absorption line observed during X-ray bursts.

If the neutron stars were born with a mass near  $1.4M_{\odot}$ , since these LMXBs are believed to have been accreting matter at a fraction of the Eddington limit for the past  $10^8$  years, their current mass should be about  $2.0M_{\odot}$ , where we have assumed an accretion energy conversion efficiency of 10%, and an average accretion rate of 10% Eddington. Therefore we conclude that it is quite reasonable to have a mass of  $2.0M_{\odot}$  for the neutron stars in LMXBs, as we have estimated (see, e.g., van den Heuvel & Bitzaraki 1995 for discussion and references).

In summary, our proposal is based on the available data at the present. The implicit assumption in making this proposal is that all the neutron stars in these LMXBs are very similar. They would all appear to have a mass, within about of  $0.2M_{\odot}$ , of  $2.0M_{\odot}$  or their mass-radius combine to give a very similar highest Kepler frequency. Our arguments appear to strongly support the sonic point model put forth by Miller, Lamb, and Psaltis (1996). In this

model, there is a clear maximum frequency for the QPO. It is either the orbital frequency at the stellar surface or the marginally stable orbit in case the star is inside the marginally stable orbit. We think the highest frequencies discussed in this letter are most likely the orbital frequencies of the marginally stable orbits.

We conclude by pointing out that it now appears possible to measure the neutron star equation of state by combining the neutron star spin frequencies ( $\nu_{ns}$ ) measured during X-ray bursts and the mass estimates ( $M_{ns}$ ) using the highest QPO frequencies. A neutron star in the process of accreting matter to spin itself up will trace a path in the  $\nu_{ns} - M_{ns}$  plane which is determined by its equation of state. If we assume that the neutron stars in LMXBs were born with the same mass and with similar or slow spin frequencies, their measured  $\nu_{ns}$  and  $M_{ns}$  will presumably form a well-defined curve, reflecting that they may have started accreting at different times and have been accreting with different average accretion rates (see, e.g., Lipunov 1992). We realize that there may be many complicating factors that can muddle this simplistic picture, nevertheless we think it represents a potentially very fruitful investigation. We also point out that it would be of great interest to study the continuum of the FFT power spectrum to the highest frequency possible. If our proposal outlined in this paper is correct, we should expect all these sources will have a feature in their power spectra at  $\sim 1100$  Hz. Searching for this feature or for the absolute maximum QPO frequency for each source will help verify our proposal and will quantitatively measure the neutron star masses.

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Table 1: Highest observed QPO frequencies from the eight sources. Note that the *RXTE/PCA Count Rate* in this table is obtained by scaling the RXTE/ASM light curves of these sources over the period between February and October 1996. The sources are arranged according to their highest observed QPO frequencies.

Source	Highest Obs. Frequency (Hz)	RXTE/PCA Count Rate (cps)	Reference
4U 1636-53	1171	1,000 to 3,000	van der Klis <i>et al.</i> 1996c.
4U 1728-34	1150	500 to 3,000	Strohmayer <i>et al.</i> 1996.
4U 1735-44	1149	800 to 4,000	Wijnands <i>et al.</i> 1996.
4U 0614+091	1145	300 to 1,200	Ford <i>et al.</i> 1996.
Sco X-1	1130	70,000 to 120,000	van der Klis <i>et al.</i> 1996a.
4U 1820-30	1066	1,000 to 5,000	Smale <i>et al.</i> 1996.
4U 1608-52	890	300 to 3,500	Berger <i>et al.</i> 1996.
GX 5-1	895	7,000 to 14,000	van der Klis <i>et al.</i> 1996d.